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Experimental study of the convection in a rotating tangent cylinder

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This paper experimentally investigates the convection in a rapidly rotating Tangent Cylinder (TC), for Ekman numbers down to $E = 3.36 \times 10^{-6}$. The apparatus consists of a hemispherical fluid vessel heated in its centre by a protruding heating element of cylindrical shape. The resulting convection that develops above the heater, i.e. within the TC, is shown to set in for critical Rayleigh numbers and wavenumbers respectively scaling as $Ra_c \sim E^{-4/3}$ and $a_c \sim E^{-1/3}$ with the Ekman number E . Although exhibiting the same exponents as for plane rotating convection, these laws reflect much larger convective plumes at onset. The structure and dynamics of supercritical plumes are in fact closer to those found in solid rotating cylinders heated from below, suggesting that the confinement within the TC induced by the Taylor-Proudman constraint influences convection in a similar way as solid walls would do. There is a further similarity in that the critical modes in the TC all exhibit a slow retrograde precession at onset. In supercritical regimes, the precession evolves into a thermal wind with a complex structure featuring retrograde rotation at high latitude and either prograde or retrograde rotation at low latitude (close to the heater), depending on the criticality and the Ekman number. The intensity of the thermal wind measured by the Rossby number Ro scales as $Ro \simeq 5.33(Ra_q^*)^{0.51}$ with the Rayleigh number based on the heat flux $Ra_q^* \in [10^{-9}, 10^{-6}]$. This scaling is in agreement with heuristic predictions and previous experiments where the thermal wind is determined by the azimuthal curl of the balance between the Coriolis force and buoyancy. Within the range $Ra \in [2 \times 10^7, 10^9]$ which we explored, we also observe a transition in the heat transfer through the TC from a diffusivity-free regime where $Nu \simeq 0.38E^2Ra^{1.58}$ to a rotation-independent regime where $Nu \simeq 0.2Ra^{0.33}$.

Key words: Rapidly rotating convection, Earth's liquid core, Tangent Cylinder, Proudman-Taylor constraint, quasi-geostrophic flows.

1. Introduction

This paper is concerned with convective flows confined in a cylindrical region by the action of the Taylor-Proudman (TP) constraint due to background rotation. Our prime motivation comes from the study of liquid planetary cores such as the Earth's. The Earth's interior is structured in layers around a solid inner core mostly made of iron (radius 1200 km), surrounded by a liquid core (external radius 3500 km) where iron dominates too. A rocky mantle occupies the region surrounding the liquid core up to

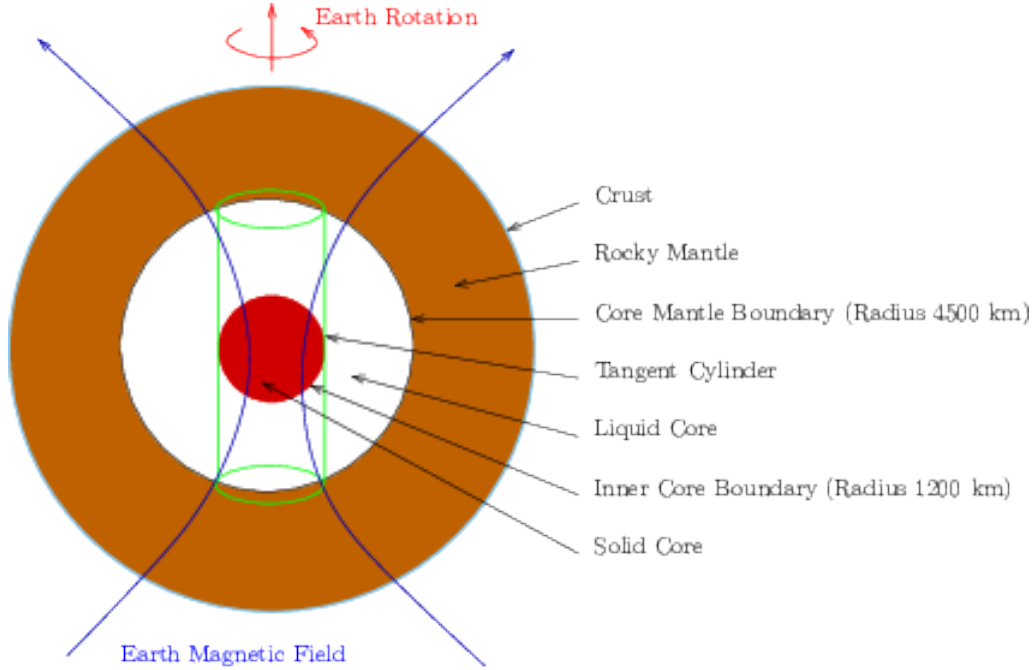


Figure 1: Schematic representation of the Earth interior. This paper focuses on the north part of the region inside the Tangent Cylinder represented in green.

the thin crust where tectonic plates assemble (see the sketch in Fig. 1). Because of the rapid rotation of the Earth (Ekman numbers down to 10^{-15}), the Taylor-Proudman (TP) constraint creates an imaginary boundary within the liquid core in the shape of a cylinder tangent to the solid inner core and extending up to the core-mantle boundary (CMB), which opposes exchange of fluid between regions inside and outside it. Consequently, the region inside the TC is subject to intense convection but also to an important effect of confinement of a different nature to that imposed by solid walls.

Until now, convection in TCs has been tackled from three different angles: linear stability, numerical simulations and experiments. Linear stability analyses mostly focused on the interplay between convection and rotation in plane geometries, where the TP constraint is absent. Early results presented in the monograph of Chandrasekhar (1961) showed that under the effect of increasing rotation, the critical Rayleigh number for the onset of convection increased as $Ra_c \sim E^{-4/3}$ while convective cells became thinner, with their wavenumber increasing as $a_c \sim E^{-1/3}$ (Here the Ekman number $E = \nu/2\Omega h^2$ is based on the background speed of rotation Ω , fluid viscosity ν and the height of the fluid layer h). A notable feature of plane rotating convection is that for a Prandtl number above a threshold value (0.677 for free-slip boundaries), the unstable mode at onset is stationary (see, e.g. Clune & Knoblauch 1993). Other theoretical approaches treated spherical shell geometries, following the early study of Busse (1970) but since convection outside the TC sets in at much lower Rayleigh numbers than inside the TC (Jones 2007), these studies mostly focused on that part of the core rather than the TC. Numerical simulations in spherical shells provided key insights into the global dynamics of the liquid core including the effect of the TP constraint, even though none was specifically dedicated to the TC. Aubert (2005) and Christensen & Aubert (2006) showed that the heat flux from the inner core boundary (ICB) to the CMB obeyed a somewhat universal scaling of

the form $Nu^* = 0.076(Ra_q^*)^{0.53}$ when expressed in terms of the heat flux-based Rayleigh number Ra_q^* and rotation-normalised Nusselt number Nu^* (see exact definitions in equations (4.6,4.7)). A similar result was obtained for the azimuthal thermal wind, which is driven by latitudinal variations in the temperature perturbation between regions inside and outside the TC: its intensity measured by a Rossby number built on the root mean square (rms) velocity scaled as $Ro = 0.85(Ra_q^*)^{0.41}$. This scaling is consistent with the hypothesis of potential vorticity conservation in a geostrophic regime (Cardin & Olson 1994) which translates into the very similar scaling of $Ro \sim (Ra_q^*)^{0.4}$. These scalings were obtained in dynamo simulations incorporating the full coupling between buoyancy, rotation and electromagnetic effects. Nevertheless, numerical simulations (Aubert 2005) showed that they remained valid with or without magnetic field. This suggests that both the heat flux and the thermal wind are mostly controlled by the amount of available buoyancy. While this amount may be altered by electromagnetic effects, their influence becomes transparent when the Rayleigh and Nusselt numbers are expressed in terms of the buoyancy itself. These scalings mostly concern regimes where rotation is dominant, and depend strongly on the level of supercriticality of convection, which controls the level of inertia acting on the flow, as well as the relative importance of rotation: where rotation does not dominate, the heat flux is expected to scale as in classical convection as $Nu \sim Ra^{0.25-0.43}$ in the range $10^5 \leq Ra \leq 10^{12}$, depending on the values of Pr and on whether the thermal or kinetic boundary layer is thickest (Grossmann & Lohse 2000). According to Julien *et al.* (2012) and Gastine *et al.* (2016), when rotation dominates, convection near the onset is governed by a balance between viscous forces, buoyancy and Coriolis forces, whereas in strongly supercritical regimes with fast rotation, the diffusivity-free regime leads to a much steeper dependence of $Nu \sim Ra^{3/2}E^2$, with a transition between these regimes controlled by parameter $RaE^{12/7}$. The transition parameters between these regimes is not specific to the TC and the question remains open as to whether they still stand when considering the dynamics of the TC only.

Aurnou *et al.* (2003) used dye visualisation to analyse convection in the only experiment dedicated to the TC to date. The authors identified convective structures, labelled "rim instabilities" that differ from those expected in plane rotating convection. Despite some data scattering, the authors suggest that the associated thermal wind was consistent with a scaling of the form $U \sim 2(B/(2\Omega))^{1/2}$, where B is the buoyancy flux. Unlike the model of Cardin & Olson (1994), this scaling reflects a balance between the Coriolis force and buoyancy outside the geostrophic regime, where the excess buoyancy and the Rossby deformation radius are set by the balance between Coriolis and buoyancy forces (Maxworthy & Narimousa 1994). Experiments and numerical simulations in a configuration similar to the TC show that this scaling is achieved in the steady state that follows the development of a baroclinic instability at the rim of the cylinder (Jacobs & Ivey 1998; Cui & Street 2001). However, a characterisation of the onset of rotating convection within the TC is currently lacking and it is not clear how convection is affected by the confinement induced by the TC.

Goldstein *et al.* (1993) and Zhong *et al.* (1993) provide an insight into the question of the confinement, in the configuration of a rotating cylinder bounded by solid walls. Linear stability analysis distinguishes slow and fast convective modes respectively localised near the centre and the lateral wall. Their occurrence depends on the aspect ratio of the cylinder, the rotation and the level of supercriticality. A precise prediction for the onset of the wall modes was later derived analytically by Zhang & Liao (2009). Importantly, in breaking translational symmetry, the presence of solid walls is shown to give rise to a retrograde (westward) precession at onset that always precludes the occurrence of the steady

modes observed in plane geometry. Nevertheless, the precession frequencies at onset are much lower than those of oscillatory convection and the associated precessing motion resembles more the retrograde thermal wind found in planetary cores than the travelling waves found at low Prandtl number in plane rotating convection (Chandrasekhar (1961); Clune & Knoblauch (1993)). Indeed, in more supercritical regimes (typically 10 times critical), experiments by Zhong *et al.* (1993) showed that the precession progressively led to a large retrograde structure centred in the middle of the cylinder. More recent studies of convection in a rotating cylinder focused on the turbulent regime (Kunnen *et al.* 2010), the influence of the boundary layers (Kunnen *et al.* 2011), of temperature-dependent fluid properties (Horn & Shishkina 2014). However, it is not clear to which extent the virtual boundaries of the TC influence the convection in the same way as the rigid boundaries of a real cylinder do; nor is it clear whether the findings of Goldstein *et al.* (1993) and Zhong *et al.* (1993), obtained for Ekman numbers of the order of 10^{-2} extend to the much lower Ekman numbers.

The purpose of this paper is to experimentally analyse convection in a Tangent Cylinder at low but experimentally accessible Ekman numbers, in view of addressing the following questions:

- (a) What are the critical conditions for the onset of convection?
- (b) Do the critical modes follow the phenomenology observed in plane or cylindrical geometries? In particular, are these modes steady?
- (c) What do these patterns evolve to in supercritical regimes?
- (d) How are heat transfer across the TC and the thermal wind affected by the combined influence of rotation and confinement within the TC?

Our experiments rely on the Little Earth Experiment (LEE) facility which we designed to reproduce rotating magnetoconvection in a tangent cylinder configuration as relevant to Earth as possible (Aujogue *et al.* 2016). Nevertheless, all experiments reported in this paper were conducted in the absence of magnetic field. The layout of the paper is as follows: in section 2, we briefly describe LEE and the measurement techniques implemented in it. In section 3, we analyse convective patterns in order to answer questions (a), (b) and (c). Section 4 is dedicated to the characterisation of the heat flux and the thermal wind while section 5 seeks to assess the confinement effect induced by the TC.

2. Experimental set-up

Our experimental apparatus is discussed in detail in Aujogue *et al.* (2016). Its main features are summarised in figure 2. We used a hemispherical dome of inner diameter $2R_D = 276$ mm filled with water or sulphuric acid. At the centre of the dome, a cylindrical heater of height 18 mm and diameter $2\eta R_D = 100$ mm protrudes into the dome. The cylindrical shape, with a horizontal heating surface, was chosen to ensure that the isothermal boundary follows an isobar when the fluid is at rest, as failure to observe this condition leads to spurious local baroclinic instabilities (Aurnou *et al.* 2003)'s. η is the ratio of heater to dome radii. Hence the thickness of the liquid corresponds to the height of fluid above the heater at the centre of the dome $d = 120$ mm. The heater provides an isothermal boundary condition at temperature T_H on its upper surface and ensures an adiabatic boundary condition at its lateral boundary. These conditions are guaranteed by the materials the heater is made of: ceramic for the upper surface and polytetrafluoroethylene (PTFE) for the lateral boundary. Indeed, considering the heat flux through

these materials (Φ_{ceramic} and Φ_{PTFE} respectively), we obtained a ratio of

$$\frac{\Phi_{\text{PTFE}}}{\Phi_{\text{ceramic}}} \sim 0.0061 \ll 1. \quad (2.1)$$

Therefore the flux through the lateral boundary can be neglected with respect to the flux through the upper surface. The heater operates as a heat exchanger fed at a constant flow rate by a heat-carrying fluid (Ethylene-Glycol) whose temperature is controlled in the static frame, so the heat flux F delivered to the working fluid inside the dome is obtained by measuring the temperature difference between the inlet and the outlet of the heater. The ceramic the upper surface of the heater is made of is SHAPAL, which has a thermal conductivity of $k_S = 92 \text{ W m}^{-1} \text{ K}^{-1}$. The corresponding Biot number, as defined by Aurnou & Olson (2001) is $Bi = 4.53 \times 10^{-4}$, which ensures a spatial temperature inhomogeneity of less than 0.1% (see Aujogue *et al.* (2016)). The temperature is monitored during each experimental run and exhibits no variations greater than the measuring uncertainty. As a consequence, the boundary condition can be considered as isothermal. Effectively, convection lowers the thermal resistance of the fluid layer by a factor Nu , so that when $Nu \gg 1$, the homogeneity of the thermal boundary condition at the heating plate is better assessed by means of a modified Biot number $Bi^* = BiNu$. The maximum value of Nu , we measured in all cases discussed in this paper was $Nu = 155$, so that Bi^* never exceeded 7×10^{-2} . This ensures that even with the most intense convection, the inhomogeneity in temperature distribution at the heater surface remained small. The temperature at the outer surface of the dome T_D is held constant by immersing the whole system in a large volume of water. Overall, the temperature difference driving convection $\Delta T = T_H - T_D$ is controlled to within $\pm 0.2^\circ\text{C}$ and spans values within $[0.7, 25]^\circ\text{C}$. The corresponding relative error on the Rayleigh number lies in the range $[1\% - 28\%]$. Indeed, as the absolute error is constant the higher the temperature, the lower is the relative error. The entire set-up including, all PIV elements is rotated about the vertical axis at angular velocity $\Omega \in [\pi, 3\pi] \text{ rad s}^{-1}$, by means of an electric motor located approximately 2.5 m below the floor of the hemispherical fluid domain. Even though we shall focus on rotating convection only in this paper, it must be kept in mind that the set-up was designed to study both convection and magnetoconvection. Hence, this choice of mechanical design, which allows us to operate the setup within the bore of large resistive magnets whilst keeping the motor away from regions of high magnetic fields. This way we aim to reproduce a geometry relevant to the Earth's Tangent Cylinder. The layout of the experiment is shown in figure 2.

The working fluids were either water or sulfuric acid (H_2SO_4) of 30% mass concentration and of respective densities $\rho_{\text{H}_2\text{O}} = 1000$ and $\rho_{\text{H}_2\text{SO}_4} = 1250 \text{ kg/m}^3$, viscosities $\nu_{\text{H}_2\text{O}} = 0.9 \times 10^{-6}$ and $\nu_{\text{H}_2\text{SO}_4} = 2.06 \times 10^{-6} \text{ m}^2/\text{s}$, thermal diffusivities $\kappa_{\text{H}_2\text{O}} = 1.4 \times 10^{-7}$ and $\kappa_{\text{H}_2\text{SO}_4} = 1.7 \times 10^{-7} \text{ m}^2/\text{s}$ at 20°C . Water was chosen for ease of use, whereas sulfuric acid was chosen as the transparent fluid with a high electric conductivity, for experiments in high magnetic fields, which were conducted at the same time. The range of parameters in which we operated the set-up is reported in table 1. The dimensionless parameters controlling the flow in the experiment are the Rayleigh number $Ra = g\alpha\Delta T d^3/\kappa$ (ratio of the buoyancy force to the viscous force), the Ekman number $E = \nu/\Omega d^2$ (ratio of the viscous force to the Coriolis force), the Prandtl number $Pr = \nu/\kappa$ (ratio of viscous to thermal diffusivities).

We measure the flow velocity with a bespoke particle image velocimetry (PIV) system in three distinct planes: one vertical plane aligned with a dome's diameter, and two horizontal planes positioned at 0.09 m and 0.02 m above the surface of the heater (see figure 2). In the former, we measure radial and axial velocities $u_r(r, z)$ and $u_z(r, z)$, where r

Control parameters	Water/H ₂ SO ₄	Aurnou <i>et al.</i> (2003)	Earth core
$E = \nu/\Omega d^2$	$4.51 \times 10^{-5} - 1.25 \times 10^{-6}$	$9 \times 10^{-4} - 10^{-5}$	10^{-15}
$Ra = g\alpha\Delta T d^3/\kappa\nu$	$1.4 \times 10^7 - 2.93 \times 10^9$	$3 \times 10^6 - 3 \times 10^{10}$	10^{22}
$Pr = \nu/\kappa$	7/12	7	10^{-2}
$\eta = R/R_D$	0.355	0.33	0.35
$Fr = \Omega^2 R/g$	0.01–0.27	0.002–0.27	4×10^{-4}
$\Delta T [^\circ\text{C}]$	[0.7–25]	< 20	6×10^3
$\Omega [\text{rad/s}]$	$[\pi/2 - 4 \times \pi]$	$[0.3 \times \pi - 1.3 \times \pi]$	7.27×10^{-5}
$D [\text{m}]$	0.1	0.1	2.44×10^3

Table 1: Range of achievable parameters in the experiment and comparison with Aurnou *et al.* (2003)’s apparatus and Earth’s core parameters. Here ν is the viscosity, Ω the rotation rate, d is the height of fluid above the centre of the heater, g the gravitational constant, α the expansion coefficient, ΔT the temperature difference between the heater and the dome, κ the thermal diffusivity, D_D the diameter of the dome and D the diameter of the heater. Note that the values of Ra for the Earth are highly uncertain (Schubert & Soderlund (2011)). (Material originally presented in Aujogue *et al.* (2016)). Values of Ω , ΔT and Fr were not provided in Aurnou *et al.* (2003) and have been calculated from dimensional and non-dimensional parameters available in this paper.

and z are respectively the radial and axial coordinates, whereas we measure radial and azimuthal velocities $u_r(r, \theta)$ and $u_\theta(r, \theta)$ in the latter two planes (Here and for the remainder of the paper, we use cylindrical coordinates with the origin at the centre of the heater’s upper circular surface). We used silver-coated particles of diameter $15\mu\text{m}$. A 500 mW continuous LASER diode generates planes of 3 mm thickness and the images were acquired at 20 frame per second with a camera of resolution 1280×1024 pixels. The corresponding spatial and temporal resolutions are respectively 0.05 seconds and 0.2 mm. This system enabled us to measure velocities within a range of $[0.002 - 0.32]\text{m/s}$, with a relative error of $[5\% - 10\%]$ on both velocity components.

Temperatures are monitored in real time using 4 K-type thermocouples: one is placed on the outer surface of the dome, one is embedded within the top surface of the heater, and two respectively measure temperature of the heat-carrying fluid at the heater’s inlet and outlet, thus providing accurate monitoring of F (with a precision of $\pm 0.14 \text{ W/m}^2$). The rotating velocity is also monitored by means of an optical sensor. The technological details of the set-up and validation tests are available in Aujogue *et al.* (2016).

The experimental protocol consists of first setting a rotation rate Ω and waiting until the flow reaches a solid body rotation (typically 30 min, verified by means of PIV measurements with the heater off). We then heat the heat-carrying fluid at a prescribed temperature, until ΔT reaches a constant value. Only when the entire system has reached a statistically steady thermal and mechanical state are the output of the thermocouples and PIV data recorded.

When rotating small devices, a centrifugal acceleration can arise. If it is large enough to compete with gravitational acceleration, the combined acceleration is not vertical anymore and the resulting convection patterns can be significantly altered. The Froude number $Fr = \Omega^2 R/g$ represents the ratio of the centrifugal acceleration to gravitational

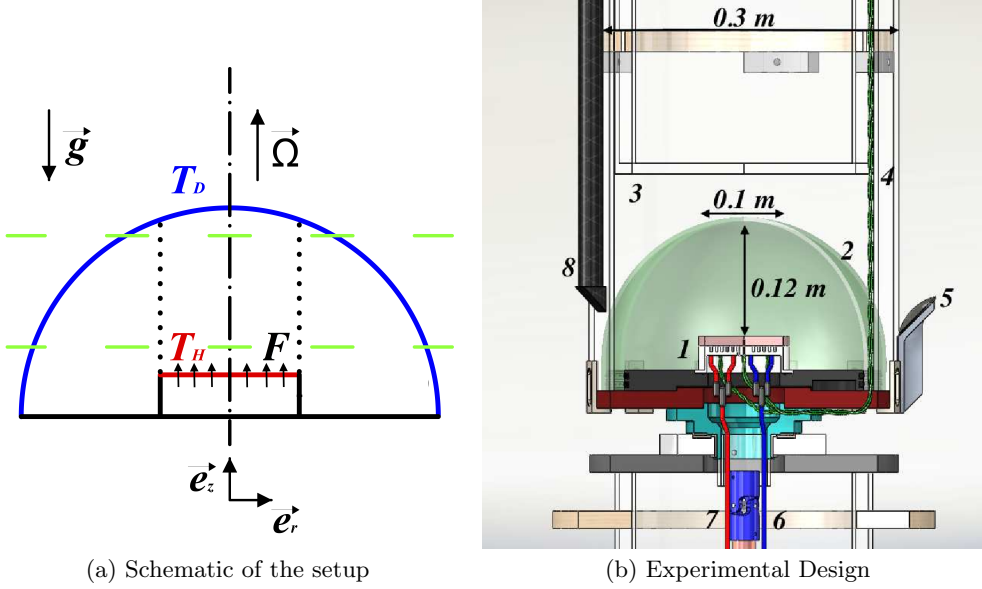


Figure 2: Left: vertical cross-section of the experimental geometry with T_H the temperature at the heater, T_D the temperature at the dome, Ω the rotation speed, g being the gravity, and F the heat flux through the heater. The green dashed lines represent the two horizontal PIV planes. The red line is the surface of the heater. The blue line is the surface of the dome. The dashed/dotted black line is the axis of symmetry and rotation of the experiment. The dotted lines are the lateral boundary of the Tangent Cylinder. Right: technical sketch of LEE, 1. Liquid Heater 2. Dome 3. Cooling Water 4. K-type thermocouples connected under and in the ceramic plate 5. Mirror 6. Torque tube 7. Pipe carrying ethylene glycol 8. PIV Camera. (Material partially presented in Aujogue *et al.* (2016)).

acceleration. For $Fr < 0.4$, convection in a solid cylinder of aspect ratio 1 at and $Pr = 7$, (close to our configuration) is not significantly different to convection at $Fr = 0$: it sets in via a pitchfork bifurcation leading to the appearance of wall modes. Fr then acts as a small imperfection parameter softening the bifurcation and slightly increases the critical Rayleigh number of every wall mode (Marques *et al.* 2007). These observations were confirmed in a cylinder of larger aspect ratio of 4 (Curbelo *et al.* 2014). For larger values of Fr , there is no base conduction state anymore and a large axisymmetric circulation driven by the centrifugal force exists, that is stable to three-dimensional perturbations. Whilst Fr is very small for planets, it is in the range $0.01 - 0.27$ in LEE. In this range, the convection is not expected to be structurally altered, and the centrifugal force may have a small stabilising influence on the onset of wall modes. In all experiments, we observed a steady base state without any evidence of the axisymmetric recirculation that would occur at higher Froude numbers, which confirms that the influence of centrifugal acceleration remained weak.

3. Structure of the convective patterns

3.1. Onset of convection

In this section, we focus on the flow near the onset of convection. The combined effect of buoyancy and rotation generates a flow structured in columns at the onset. This is illustrated by the contours of vertical velocity obtained for different values of E on Fig. 4. Note that the columns are exclusively confined within the TC. This can be inferred from the sharp drop of vertical velocity visible all along the TC boundary. It can also be seen from the radial profiles of velocity in Figs. 12a and 12c. It is noteworthy that the configuration of our experiment favours confinement within the TC because (i) the heat flux is generated along the whole horizontal section of the TC and nowhere outside it and (ii) gravity is always parallel to the TC boundary, unlike in the Earth where they intersect. Hence, the buoyancy force never directly acts as to break the TP constraint in LEE, unlike in the Earth. However, the small residual flow outside the TC is further attenuated by an increase in background rotation (see fig 12a-12d). Conversely, qualitative experiments conducted without rotation show very different picture where plumes take the entire space above the heater, for the flow to return entirely outside of the would-be TC. Considering the sharpness of the velocity jump across the TC boundary, these elements give a good evidence that the confinement we observe mainly results from TP constraint. Columns become thinner at lower values of E , a tendency that has been previously observed both in spherical and plane geometries (Sreenivasan & Jones 2006; Chandrasekhar 1961; Aujogue *et al.* 2015).

On figure 3, we show that variations of the critical Rayleigh number with E follow a scaling of $Ra_c = (32.3 \pm 4) \times E^{-1.29 \pm 0.05}$. Despite a very different geometry, this scaling is in good agreement with the theoretical prediction of $Ra_c = 22.3 \times E^{-4/3}$ for rotating convection in an infinitely extended plane layer (Chandrasekhar 1961). This result suggests that the critical Rayleigh number is not measurably affected by the geometry differences between a plane and our TC and reflects a very robust feature of plane rotating convection.

We shall now analyse the horizontal size of the convective structures present at the onset. Since translational invariance in the horizontal plane is lost in our geometry, we extract the horizontal size of convective structures by seeking the separation r_0 corresponding to the first zero of the spatial correlation function built from u_r , and averaged over time and z :

$$C_{u_r}(\delta r) = \left\langle \int_{\mathcal{V}} u_r(r + \delta r) u_r(r) dr dz \right\rangle_t, \quad (3.1)$$

where \mathcal{V} represents the intersection of the meridional plane lit by the PIV laser and the region inside the TC. At the onset, the associated wavenumber $a_c = 2\pi R/r_0$ can be compared to the critical wavenumber predicted for the onset of rotating convection in an infinite plane layer (Fig. 3). In the TC geometry, we find a scaling of

$$a_c = (0.58 \pm 0.08) \times E^{-0.32 \pm 0.05} \quad (3.2)$$

when the plane layer theory predicts $a_c = 1.65 \times E^{-1/3}$ (Aujogue *et al.* 2015). Although both structure sizes exhibit the same scaling exponent, critical wave numbers are significantly lower in the TC geometry than for the infinite plane layer. We shall see in section 3.2 that the reason for this discrepancy originates in the topological structure of the critical modes.

Lastly, a remarkable feature of the onset of rotating convection in plane layers is that for the values of Pr considered in this paper, linear stability predicts a steady critical mode (Clune & Knoblauch 1993). By contrast, in all our measurements, we found a

time-dependent flow at the onset. Inspection of the flow in horizontal planes reveals that the convective plumes are subject to a slow retrograde precession. The variations of the corresponding frequency ω_p derived from the maximum velocity along θ in the horizontal plane and normalised by the background angular frequency are reported in figure 7. The precession at the onset of convection and beyond has been studied theoretically and experimentally in Goldstein *et al.* (1993) and Zhong *et al.* (1993) in rigid rotating cylinders of various aspect ratios. In these studies, the authors showed that the loss of translational symmetry in the radial direction necessarily induced a precession in the critical mode and that the corresponding frequency normalised by the background rotation Ω , ω_{p0} obeyed a scaling of the form $\omega_{p0} = \delta E^{-1}$. Experiments by Ecke *et al.* (1992) determined a value of $\delta = 0.1$ for a radius-to-height aspect ratio of $\Gamma = 1$. Our measurements produce a value of $\delta = 0.07 \pm 0.005$. This value is close to that found by Ecke *et al.* (1992), despite a lower aspect ratio (measured at the centre of the heater) of $\Gamma = R/d = 5/12$.

The precession of convective structures at the onset in a cylindrical geometries is normally associated with the onset of modes that are localised either at the centre of the cylinder or near the wall, respectively centre modes and wall modes (Goldstein *et al.* 1993). The values of the precessing frequencies and of δ we find point to wall modes rather than centre modes (Ecke *et al.* 1992). Nevertheless, in cylinders bounded by solid walls, the onset of wall modes normally takes places at a lower Rayleigh number (denoted Ra_W) than the steady modes that ignites plane convection for the same value of E . Zhang & Liao (2009) provide an estimate for Ra_W and the corresponding azimuthal wavenumber a_W , which are reported on Fig. 3 and expressed in our notations and for the present configuration as

$$Ra_W = 31.81E^{-1} + 46.49E^{-2/3}, \quad (3.3)$$

$$a_W = 2.118 - 12.15E^{1/3}. \quad (3.4)$$

The fact that in the TC, Ra_c follows the scaling for plane convection rather than that of wall modes is perhaps explained by the fact that unlike a solid wall, the Taylor-Proudman constraint is absent when the fluid is at rest. Wall modes would be expected to occur at a lower critical Rayleigh number than the onset of plane convection. Since, however, no confinement exists at such Rayleigh numbers, the mechanism for their onset is absent. On the other hand, as soon as convection starts, *i.e.* for Rayleigh numbers close to those for which the plane layer would be unstable, the TP constraint becomes active and selects modes that are closer to those observed in a cylinder than those observed between infinite planes. Hence, the TP constraint may not influence the flow right at the onset, but only once unstable structure have sufficiently developed.

In summary, the study of critical Rayleigh, plume size, and precession frequency shows that the onset of convection in a TC corresponds to a hybrid behaviour between those of convection in an infinite plane layer and in a rotating cylinder. Certainly, the scaling for the critical Rayleigh number from the plane layer theory is reproduced in the TC but size and time-dependence of the flow are to some extent more accurately described by the phenomenology of convection in rigid rotating cylinders.

3.2. Supercritical flow patterns

We shall now explore the evolution of the patterns from the onset of convection into the supercritical regime, focusing on two values of the Ekman number $E = 1.15 \times 10^{-5}$ and $E = 6.36 \times 10^{-6}$, for critical parameters $R_c = (Ra - Ra_c)/Ra_c$ in the range 0.1–13. Figures 5a to 5f illustrate the development of convective patterns for a criticality in the range $0.1 \leq R_c < 12$ and $E = 1.15 \times 10^{-5}$. Near the onset of convection ($R_c = 0.13$, figure

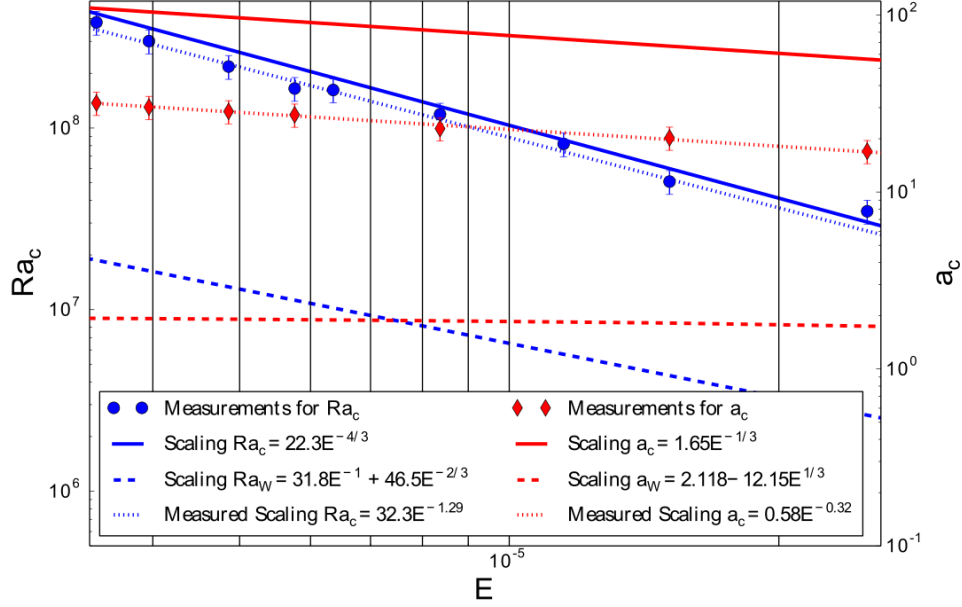


Figure 3: Critical Rayleigh number Ra_c and critical wave number $a_c = 2\pi R/r_0$ at onset *vs.* the Ekman number E for three geometries: present case (symbols and dotted lines), infinite plane layer (solid lines) and wall modes in a finite cylinder obtained from Zhang & Liao (2009)'s theory (Eqs. (3.3 and 3.4), dashed lines).

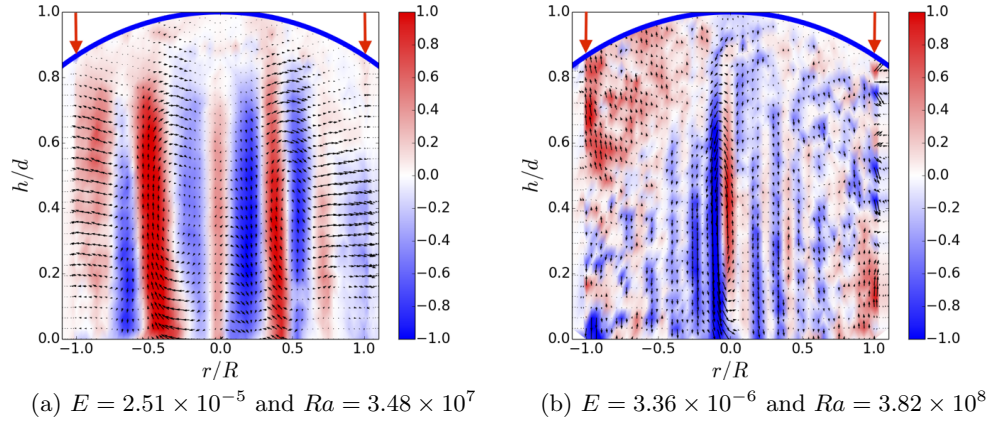


Figure 4: Average over time of the vertical component of velocity from PIV measurement above the liquid heater at the onset of convection. The averaging time is 15 times smaller than the precession timescale $\tau_p = 2\pi/\omega_p$. Velocities are normalised by the maximum velocity of the flow. Here z and r are normalised by the maximum height above the heater and the radius of the heater respectively. The blue line represents the inner boundary of the glass dome. The red arrows point to the position of the TC.

5a) we observe a behaviour found in the linear stability analysis performed by Goldstein *et al.* (1993) on a rapidly rotating cylinder with non-slip boundary conditions at the top and bottom and adiabatic boundary conditions on the side walls. In this study, the authors characterised two different types of convective mode at onset, labelled fast and slow. The fast modes correspond to motion at the edge of the cylinder. The slow modes describe convective patterns localised at the centre of the cylinder. Goldstein *et al.* (1993) also showed that the changeover between fast and slow modes is strongly dependent on the aspect ratio of the cylinder. On figure 5a, we observe a structure similar to a fast mode of azimuthal wavenumber $m = 2$. For this wavenumber, Goldstein *et al.* (1993) predicted that the fast mode was the most unstable below an aspect ratio of 1.84, a condition satisfied in the TC geometry of our experiment. Note, however that the fast/wall mode structure is not as sharp as for higher levels of criticality. This is in part due to the technical difficulty is accurately resolving the small velocity differences near the onset. It may also reflect that the TC boundary exerts a somewhat weaker influence near the onset than the solid walls of Goldstein *et al.* (1993)'s cylinder.

For $R_c = 1.26$ (Fig. 5b), several columns are gathered around the centre of the TC. These are reminiscent of structures which Aurnou *et al.* (2003) call quasi-geostrophic modes, with the difference that in their experiment, dye visualisations suggested that they were forming on the outside of the TC. When further increasing R_c ($R_c = 4.58$; figure 5c), we see an interaction between these centre modes and "wall modes" forming near the side boundary. On figures 5d and 5e, we observe that for $R_c \geq 6.29$ the centre modes are merging into one larger structure, which evolves into a large central, retrograde vortex in the last stage of the convection observed in the experiment (figure 5f, $R_c = 11.27$). The evolution towards a central retrograde vortex for critical parameters exceeding $R_c = 10$ was also found by Zhong *et al.* (1993) in experiments on convection in a rotating cylinder of aspect ratio $\Gamma = 1$ (*i.e.* wider than in the present case).

Figures 6a to 6f show the flow at a smaller Ekman number, $E = 6.36 \times 10^{-6}$, for $R_c \in [0.35 \times Ra_c, 11.91 \times Ra_c]$. Near onset ($R_c = 0.35$, figure 6a), the convective patterns are much smaller than for a comparable criticality at $E = 1.15 \times 10^{-5}$. When the criticality increases, we observe modes that correspond to the modulated modes described by Goldstein *et al.* (1993). These feature spiralling arms (Fig. 6b) and were also observed experimentally by Zhong *et al.* (1993) for a comparable level of criticality ($R_c = 2.56$). Zhong *et al.* (1993) showed that these spiralling patterns arise from the outer wall of the cylinder as azimuthal mode. In the TC, we observe such an azimuthal mode with a corresponding wavenumber $m = 2$. Such large differences between supercritical patterns at different Ekman numbers were theoretically predicted by Goldstein *et al.* (1993), whose analysis shows that when E is varied, the lowest critical Rayleigh number is alternatively achieved by either a fast or a slow precessing mode. At higher criticality, convective patterns reflect a combination of wall modes and centre modes (at $R_c = 2.32, 4.40, 9.17$ on figures 6c, 6d and 6e), as for $E = 1.15 \times 10^{-5}$. Similarly to the more slowly rotating case, convective patterns converge towards a central retrograde structure at the highest levels of supercriticality explored here (Fig. 6f).

For both values of E , the main features of the flow patterns (alternative presence of fast and slow modes near the onset of convection depending on E , evolution towards a large retrograde vortex for $R_c \gtrsim 10$) support the view that the convection in the TC behaves as convection in a solid cylinder rather than in an infinite plane layer. It is also interesting to note that the flow patterns discovered by Goldstein *et al.* (1993) and Zhong *et al.* (1993) at relatively high values of E ($\sim 10^{-3} - 10^{-2}$) remain dominant at

the much lower values explored in our experiment ($\sim 10^{-6} - 10^{-5}$). Such robustness may indicate that convection is in an asymptotic regime of rotation as far as flow structures are concerned and that similar structures might also be found in regimes of even more rapid rotation, such as the Earth's.

It should also be mentioned that other authors mention the existence of instabilities at the rim of the TC (see Cui & Street (2001); Aurnou *et al.* (2003)). While it is difficult to distinguish such instabilities from convective instabilities within the cylinder from visualisations only, Maxworthy & Narimousa (1994) found that such instabilities occur for a Rossby number based on vertical motion $Ro^* = u_z/2\Omega$ greater than 0.28. This parameter remains one or two orders of magnitude below this value in most of our experiments, and below 0.1 in the most supercritical cases (for $R_c \simeq 10$). This is a good indication that the structures we observe are convective patterns, rather than rim instabilities.

3.3. Evolution of the plume size and precession frequencies in supercritical regimes

The succession of supercritical patterns is reflected in the evolution of the dominating wavelengths and precession frequencies with criticality R_c gathered on Fig. 7. We find that beyond the onset of convection the dominating wavenumber follows a scaling of the form

$$aE^{1/3} = (0.5 \pm 0.025) \times (R_c + 1)^{-0.45 \pm 0.05}. \quad (3.5)$$

This scaling extends the scaling for $a_c(E)$ found at the onset of convection to supercritical regimes. It implies that as convection becomes more intense, the flow rearranges itself with fewer larger structures that are more efficient to carry the heat flux across the TC up to the point where only one structure is left for $R_c \gtrsim 10$.

The precession frequency first sharply increases in the weakly supercritical regime and subsequently saturates. It is nevertheless difficult to tell whether an asymptotic value is reached in the limit $R_c \rightarrow \infty$. Using a Landau model for the bifurcation, Goldstein *et al.* (1993) showed that in weakly supercritical regime, the precession normalised by Ω should vary as:

$$\omega_p = E^{-1}(\delta - \phi R_c) + O(R_c^2), \quad (3.6)$$

where the values of constants δ and ϕ depend on the aspect ratio of the cylinder (with $\delta \rightarrow 0$ in the limit of large aspect ratio to recover the stationary onset of convection in an infinitely extended plane layer, see Chandrasekhar (1961)). The values of $\delta = 0.1$ and $\phi = 5$ were obtained experimentally by Zhong *et al.* (1993) and Ecke *et al.* (1992) for a cylinder of aspect ratio $\Gamma = 1$ and values of E greater than 10^{-3} . The variations of $\omega_p(r_c)$ extracted from experiments at several Ekman numbers for $R_c > 0$ are reported in Fig. 7. They are best fitted over the widest measured range of values by an exponential law of the form:

$$\omega_p = (3 \pm 0.5) \times E^{-1}(1 - e^{(-0.24 \pm 0.01)R_c}) + 0.1 \times E^{-1}, \quad (3.7)$$

Expanding (3.7) to $\mathcal{O}(R_c)$ and identifying with (3.6), we find that $\phi = 0.534 \pm 0.105$. The value of δ is consistent with the findings of Ecke *et al.* (1992), but the precession frequency appears to vary significantly less in the supercritical regime in the present case than in the case of a solid rotating cylinder. It is, however, difficult to attribute this difference to any of the factors that differ between the two problems: the difference in shape of the upper domain boundary, a different aspect ratio, a three-order of magnitude difference in the Ekman number, different mechanical and thermal boundary conditions at the lateral boundary of the cylinder. Nevertheless, the precessing motion at onset and in supercritical regimes adds further support to the view that the phenomenology

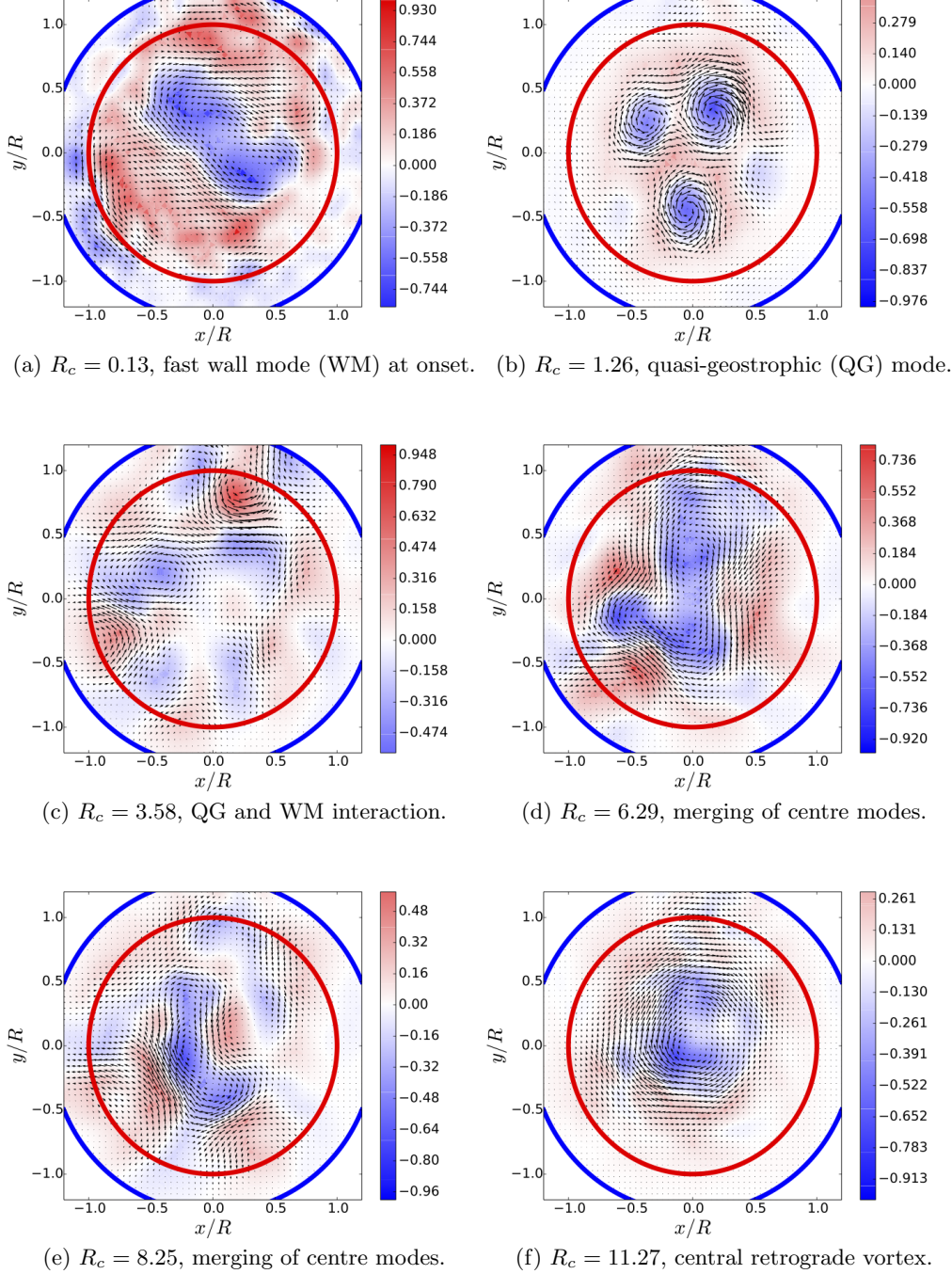


Figure 5: Time averaged velocity fields (arrows) with vorticity field (colorbar) in the horizontal plane at $z/d = 3/4$ for different Rayleigh numbers with $E = 1.15 \times 10^{-5}$. The blue line represents the boundary of the glass dome. The red line represents the position of the heater that defines the TC. Wall modes can be identified where vorticity extrema are present near the boundary of the TC (e.g. 5a, 5c). The averaging time is 15 times smaller than the precession timescale $\tau_p = 2\pi/\omega_p$.

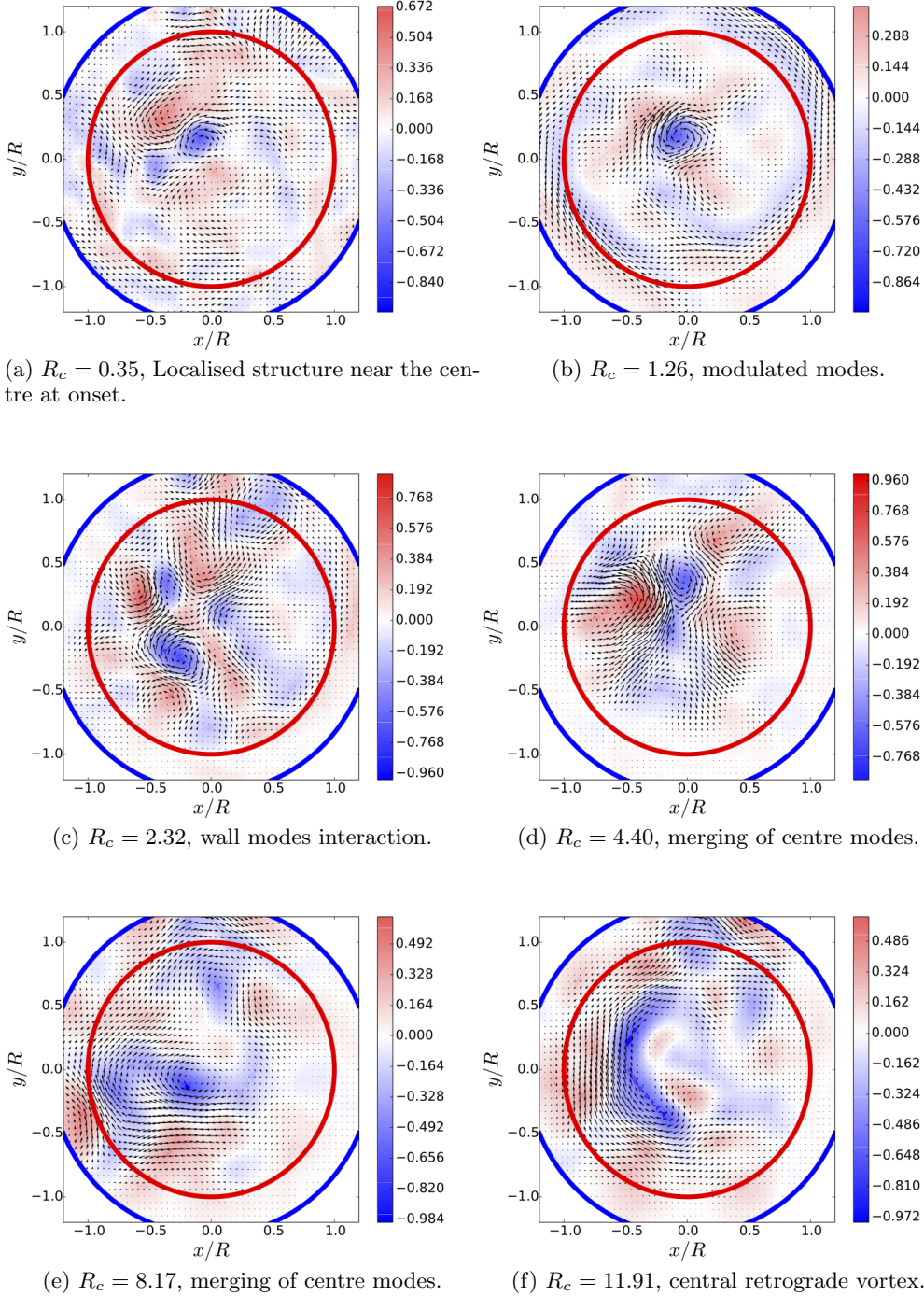


Figure 6: Time averaged velocity fields (arrows) with vorticity field (colorbar) in the horizontal plane at $z/d = 3/4$ for different Rayleigh numbers with $E = 6.36 \times 10^{-6}$. The blue line represents the boundary of the glass dome. The red line represents the position of the heater that defines the TC. Wall modes can be identified where vorticity extrema are present near the boundary of the TC (e.g. 6c). The averaging time is 15 times smaller than the precession timescale $\tau_p = 2\pi/\omega_p$.

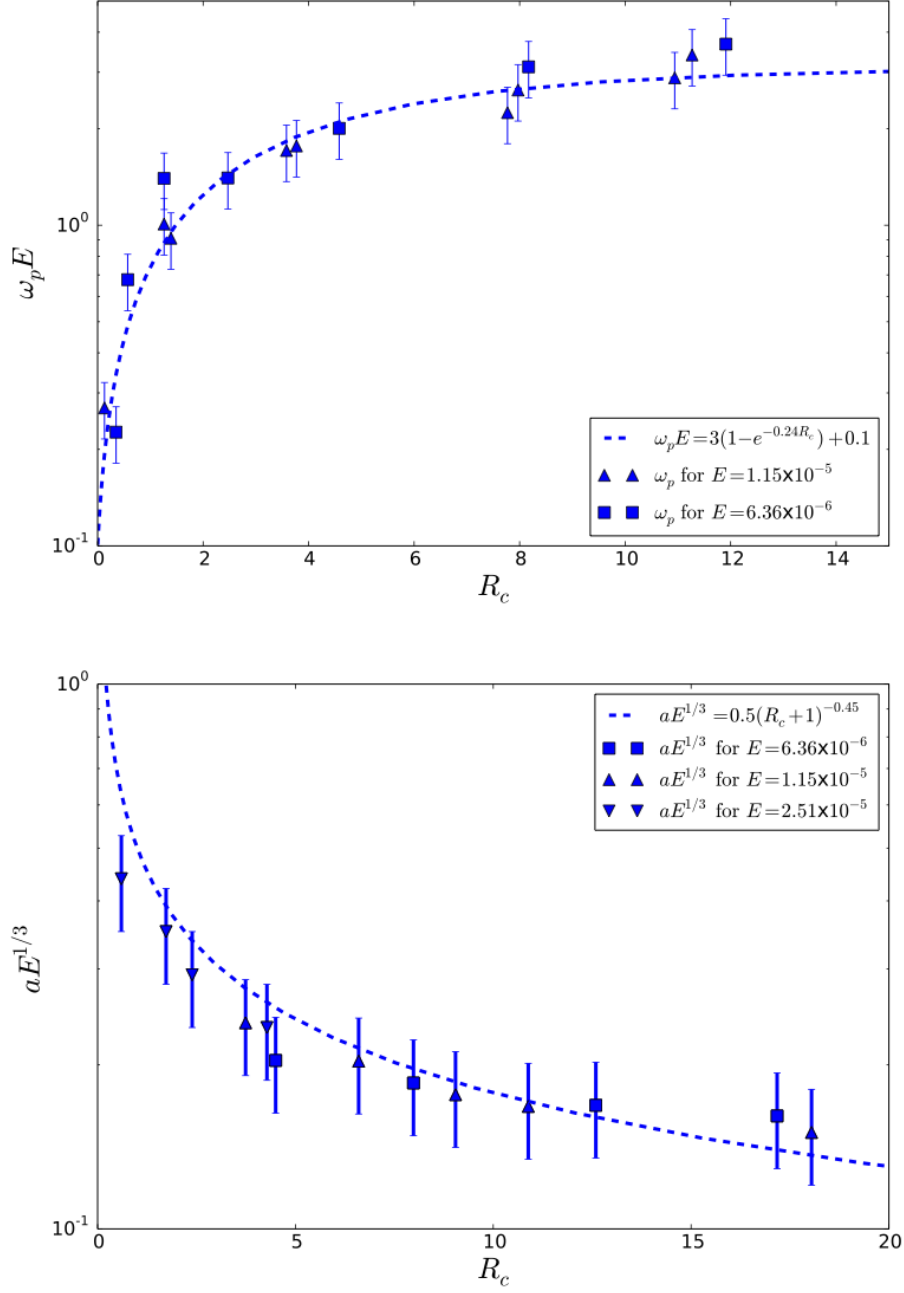


Figure 7: Evolution of the precession ω_p and the wave number a as function of the degree of criticality. Raw and interpolation data for this figure are available as the supplementary material.

of convection in the tangent cylinder obeys the same mechanisms as in a solid rotating cylinder, both at the onset and in supercritical regimes.

4. Heat flux and thermal wind

4.1. Heat Flux through the Tangent Cylinder

The efficiency of convection is best measured by its ability to transport heat through a fluid layer. Here we characterise the variations of the heat flux with Ra and E by means of the Nusselt number which represents the ratio of the heat flux in the fluid to the purely conductive heat flux:

$$Nu = \frac{Fd}{k\Delta T}, \quad (4.1)$$

where we recall that F is the heat flux through the top surface of the heater, obtained from the difference between the temperature of the heat-carrying fluid at the inlet and outlet of the heater and d is the height of fluid above the heater. The variations of Nu with Ra are reported in Fig. 9 for several values of E . Since F is measured at the heater, it reflects the total heat transferred across the entire dome. As all of the flux transits through the TC, F is an upper bound for the total heat flux through the upper surface of the TC because thermal losses occur at the lateral boundary of the heater. Given the low values of the velocity near that boundary, these losses are mostly likely predominantly of conductive nature, especially at low criticality. Two regions of parameters clearly appear. In the large Ra limit, the heat transfer becomes independent of the rotation and follow a scaling of the form (see top graph):

$$Nu = (0.2 \pm 0.04) Ra^{0.33 \pm 0.03}. \quad (4.2)$$

This suggests that in these strongly supercritical regimes, heat transfer are not influenced by the Coriolis force anymore. Indeed, in the range of Rayleigh and Prandtl numbers we considered, similarly low exponents of Ra in the range $1/4$ - $3/7$ are expected for plane non-rotating Rayleigh-Benard convection (Grossmann & Lohse 2000) and in spherical shells with radial gravity (Gastine *et al.* 2015). At moderate Rayleigh numbers, by contrast, ($Ra \gtrsim 8 \times 10^8$ for $E = 1.15 \times 10^{-5}$, $Ra \gtrsim 1 \times 10^9$ for $E = 6.35 \times 10^{-6}$ and $Ra \gtrsim 1.5 \times 10^9$ for $E = 4.46 \times 10^{-6}$, see bottom graph), Nu exhibits a strong dependence on E as well as Ra , of the form:

$$Nu = (0.38 \pm 0.02) \times Ra^{1.58 \pm 0.06} E^{2 \pm 0.04}. \quad (4.3)$$

This law is very close to the theoretical scaling of $Nu \sim 0.15 Ra^{3/2} E^2$ put forward by Julien *et al.* (2012) (see also Gastine *et al.* (2016)) as a signature of the diffusivity free-regime. These authors found that this scaling was verified in rotating convection in a spherical shell with radial gravity for a range of values of Ra and E that is essentially the same as its range of validity in LEE. Their theoretical scaling argument, however, does not invoke geometry nor the orientation of the buoyancy force. It relies on the assumptions that (i) the largest contribution or the temperature gradient originates in the bulk and not in boundary layers, (ii) in the limit of no rotation, the ultimate regime of classical Rayleigh-Benard convection is recovered (Kraichnan 1962), (iii) in the limit where rotation dominates inertia, the heat flux solely depends on R_c King *et al.* (2012). Hence, this scaling can be reasonably interpreted as the signature of the diffusivity-free regime of rotationally-dominant convection in the TC configuration too. Experiments and numerical simulations on convection in a rotating cylinder filled with water (Cheng *et al.* 2015; King *et al.* 2012) also exhibit a similar transition between steep variations

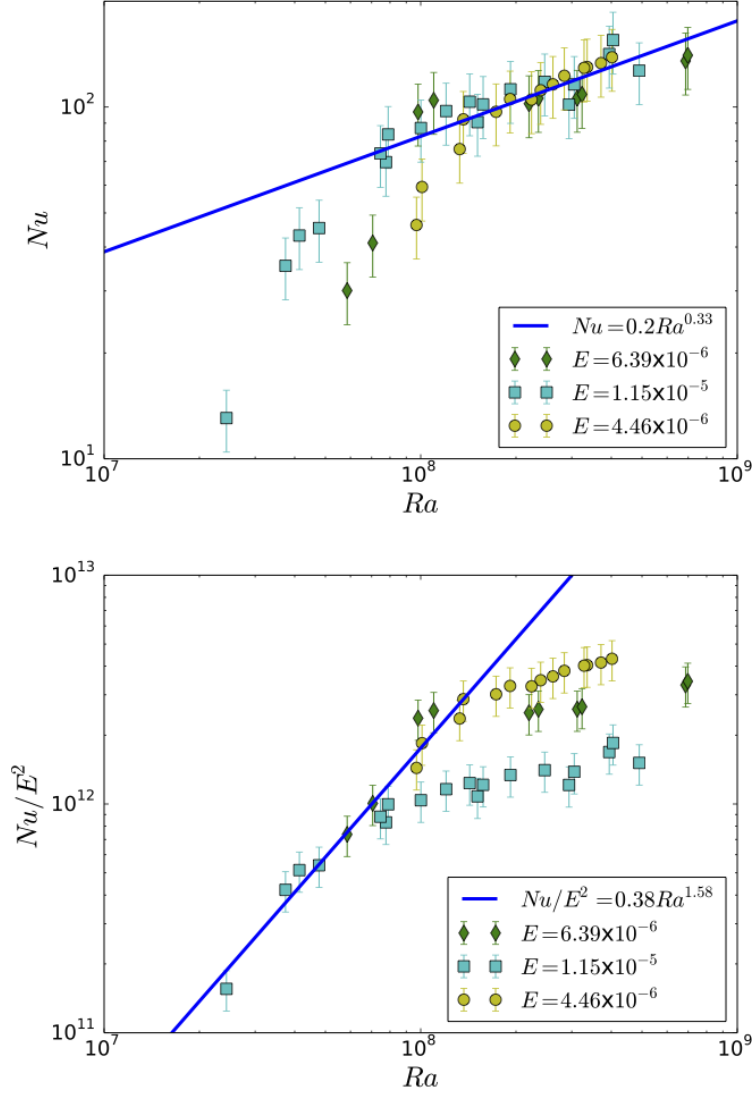


Figure 8: Variations of the Nusselt number N emphasising the scaling $Nu/E^2 \simeq 0.2Ra^{0.33}$ for large Rayleigh numbers (top) and the the scaling $Nu \simeq 0.38E^2Ra^{1.58}$ at moderate Rayleigh numbers (bottom). Raw and interpolation data for this figure are available as the supplementary material.

of $Nu(Ra)$ in the weakly supercritical regime and a more a strongly supercritical regime where $Nu \sim Ra^{0.32}$ that is very close to (4.2). On the other hand, the scaling law at low Ra , was found to strongly depend on E , with an exponent varying from $Nu \sim (RaE^{4/3})^{6/5}$ at low Ekman numbers to $Nu \sim (RaE^{4/3})^{3.6}$ (for $E = 10^{-7}$).

An alternative but equivalent way to analyse the variations of the heat flux consists of seeking how flux normalised by rotation varies with the Rayleigh number based on the heat flux itself. One advantage of this approach is to quantify convection in terms of the available buoyancy. Buoyancy may indeed result from a heat flux at the boundary

but also from a solute mass flux, as in the core of the Earth and other planets. On this basis, we follow Aubert *et al.* (2001); Aubert (2005); Christensen & Aubert (2006) and introduce the modified Nusselt number, Rayleigh number and a modified diffusionless Rayleigh number based on the heat flux, respectively defined as

$$Nu^* = Nu \times E \times Pr^{-1}, \quad (4.4)$$

$$Ra^* = Ra \times E^2 \times Pr^{-1}, \quad (4.5)$$

$$Ra_q^* = Ra^* Nu^*. \quad (4.6)$$

Cheng & Aurnou (2016) stresses that scalings of the form $Nu \sim Ra^\alpha$ and $Nu^* \sim Ra_q^{*\beta}$ satisfied $\beta = \alpha/(1+\alpha)$. Here $\alpha = 0.33 \pm 0.03$, $\alpha/(1+\alpha) \in [0.23, 0.27]$ and $\beta = 0.3$. Hence, these scalings offer an alternative way of representing the data from figure 9, rather than new data.

Christensen (2002) found that these quantities obeyed a scaling law of the form $Nu^* \sim (Ra_q^*)^{5/9}$.

This result was obtained with numerical simulations in a spherical shell geometry for $Ra_q^* \in [10^{-7}, 10^{-3}]$. Aurnou (2007) identified this scaling as indicative of a rapidly rotating regime. Further, using the data of Sumita & Olson (2003), it was shown that for low values of Ra_q^* the relation between Nu^* and Ra_q^* was better fitted with a power law of the form $Nu^* \sim (Ra_q^*)^{0.29}$. In the present configuration, Ra_q^* varies between 10^{-10} and 10^{-6} and therefore falls within a similar range to the experiments of Sumita & Olson (2003). Collapsed data reported in Fig. 9 show that points in the regime following the $Nu \sim Ra^{0.33}$ law obey a scaling close to that found by these authors:

$$Nu^* = (0.0046 \pm 0.0005) \times (Ra_q^{*1 \pm 0.03})^{0.26 \pm 0.04}. \quad (4.7)$$

The exponent being closer to 0.29 than 5/9 confirms that this regime is one the flow is outside the quasi-geostrophic regime, where convection-driven inertia plays an important role.

4.2. Thermal wind

In axisymmetric geometry, the balance between buoyancy and Coriolis forces gives rise to azimuthal motion, seen through the balance between the curl of these forces:

$$\frac{\partial u_\theta}{\partial z} \sim -\frac{g\beta}{2\Omega} \frac{\partial T'}{\partial r}, \quad (4.8)$$

where β is the coefficient of thermal expansion and T' is the temperature perturbation. The strongest retrograde motion is found at the higher latitude, such as those of our PIV planes represented in figures 5 and 6, located at a latitude of 51.5° (or, equivalently at $z/d = 0.75$ above the heater).

On the other hand, the direction of the thermal wind may reverse at low latitude. Hence, we extract from these measurements a radial profile of the azimuthal wind at two different latitudes (see figure 2) by means of an azimuthal and time average of the azimuthal velocity $\langle u_\theta(r, z) \rangle_{\theta t}$: additionally to the high latitude,

the second latitude we consider corresponds to a plane close to the heater surface (20° , or, equivalently at $z/d = 0.167$ above the heater).

Results are plotted on figures 10a to

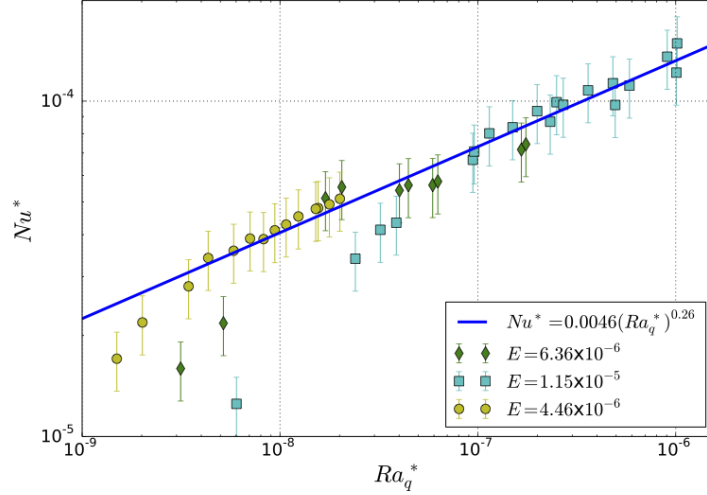


Figure 9: Variations of Nu^* with Ra_q^* . Raw and interpolation data for this figure are available as the supplementary material.

10d.

At high latitude (figures 10a and 10b), the profiles exhibit a strong negative maximum followed by slightly positive values at larger radii, corresponding to a strong retrograde motion surrounded by a slightly prograde motion. Near the onset of convection, some prograde motion exists near the centre, that disappears as the flow becomes more supercritical. Despite the succession of different patterns observed in the supercritical regime, the intensity of the retrograde motion steadily increases with criticality. For strongly supercritical flows ($R_c \gtrsim 11$), velocity patterns show a single structure slightly outgrowing the TC (we shall see in section 5 that motion outside the TC does not necessarily imply that the Taylor-Proudman constraint is broken). This phenomenon appears for slightly higher criticality at higher values of E ($R_c \gtrsim 9$ at $E = 1.15 \times 10^{-5}$ and $R_c \gtrsim 11$ at $E = 6.36 \times 10^{-6}$). In both cases, this confirms that in the most supercritical regimes, retrograde motion progressively invades the high-latitude region of the TC, where the flow becomes dominated by a large central retrograde vortex.

Profiles at low latitude, nearer the solid inner core provide a better picture of the three-dimensional structure of the thermal wind (Figures 10c and 10d). In both cases, a prograde wind first develops near the solid core at low levels of criticality and increases in intensity with R_c . At $E = 1.15 \times 10^{-5}$, the prograde wind starts weakening at the centre from $R_c = 2.37$ and starts becoming retrograde around $R_c = 9.92$. At higher rotation, weakening of the prograde wind near the solid core occurs only for $R_c > 9$ and we did not reach a regime where it reversed. This difference in levels of criticality for the weakening and reversal of the thermal wind near the solid core can be understood from the scaling for the critical Rayleigh number $Ra_c \sim E^{-4/3}$ and (4.8) which imply that $\partial_z u_\theta \propto (R_c + 1)E^{1/3}$. From this scaling, thermal wind with a given vertical gradient of u_θ is expected to occur at increasingly high levels of criticality when E is decreased. Hence, the larger velocity gradients corresponding to a thermal wind flowing in opposite directions at high and low latitudes survive at higher levels of criticality when E decreases. Interestingly, a clear azimuthal prograde jet is present at the edge of the tangent cylinder.

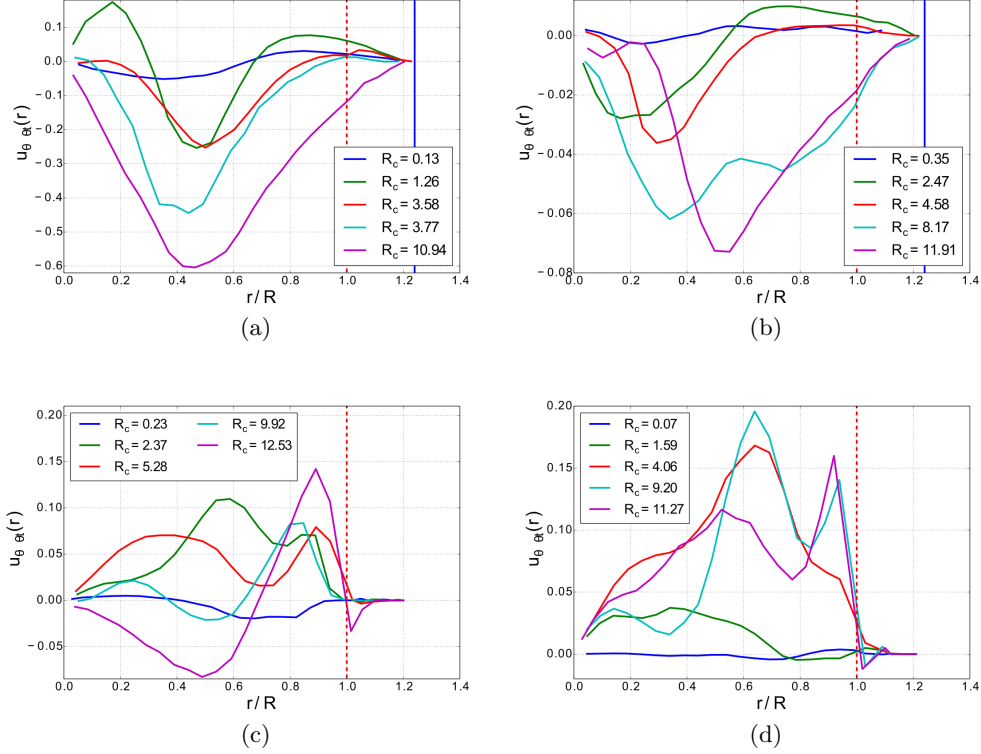


Figure 10: Azimuthally and time-averaged radial profiles of azimuthal velocities $\langle u_\theta \rangle_{\theta t}(r)$, for several levels of criticality. Top: data from horizontal plane at 51.5° . Bottom, data from horizontal plane at 20° . Left $E = 1.15 \times 10^{-5}$, Right: $E = 6.36 \times 10^{-6}$. Vertical dashed line: TC boundary, vertical solid line: position of the glass dome.

der on the inside. The jet is thinner at faster rotation. It appears for $R_c \sim 2.37$ at $E = 1.15 \times 10^{-5}$ and $R_c \sim 4.06$ at $E = 6.36 \times 10^{-6}$, and its intensity slightly increases

with criticality. One cannot but notice the similarity between this structure and the non-axisymmetric azimuthal jets near the TC inferred by Livermore *et al.* (2017) from satellite measurements of time variation of the Earth's magnetic field. Although the forces driving the jet remain unknown, the authors suggest that they are most likely of magnetic origin, an effect that is absent in the present study. Azimuthal jets were also noticed in measurements in the flow patterns inferred from measurements of the Earth magnetic field (Hulot *et al.* 2002).

To conclude the characterisation of the thermal wind, we shall quantify its intensity in terms of the Rossby number Ro , and its variations with the flux-based Rayleigh number, Ra_q^* . The Rossby number, defined as

$$Ro = \frac{U_m}{2R\Omega} \quad (4.9)$$

measures the ratio of inertia to the Coriolis force, where U_m is chosen as the maximum retrograde velocity in the averaged profiles of the thermal wind measured in the high

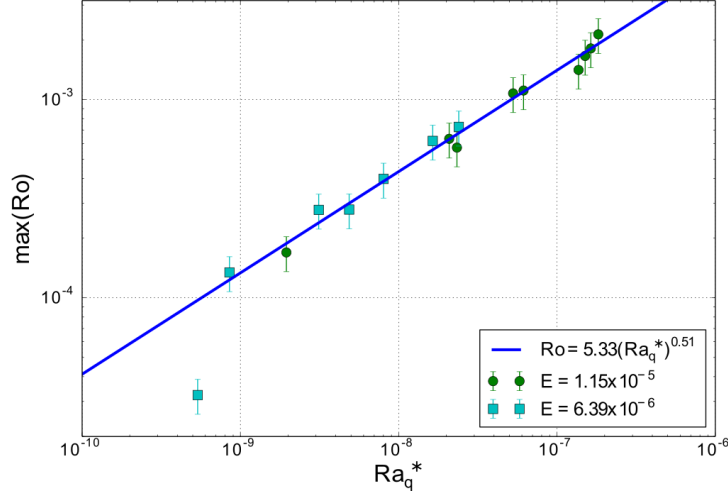


Figure 11: Variations of the Rossby number Scaling Ro with Ra_q^* . Raw and interpolation data for this figure are available as the supplementary material.

latitude plane (reported in figures 10a and 10b). Our results, reported in figure 11, obey the scaling

$$Ro = (5.33 \pm 0.3) \times (Ra_q^*)^{0.51 \pm 0.04}. \quad (4.10)$$

This scaling is very close to the scaling relating the azimuthal velocity scale U_θ and buoyancy B , $u_\theta \sim (B/\Omega)^{1/2}$ (here, $Ro \sim Ra_q^{*1/2}$) first derived theoretically by Maxworthy & Narimousa (1994) for the thermal wind in oceanic convective plumes. It is derived from the azimuthal curl of a local balance between Coriolis force and buoyancy outside the geostrophic regime, where the excess buoyancy and the Rossby deformation radius are set by the balance between Coriolis and buoyancy forces (Maxworthy & Narimousa 1994). Both Aurnou *et al.* (2003) and Aubert (2005) report the same scaling when thermal wind is present respectively in experiments in a TC geometry and in numerical simulations in full spherical shell. Hence (4.10) provides further evidence that the azimuthal motion we are detecting is thermal wind. the vertical gradients of azimuthal velocity induce a departure to geostrophy of order $\mathcal{O}(Ro)$, which is significantly smaller than the vertical velocities associated with the plumes. Hence the thermal wind may not be the main source of the ageostrophy suggested by the variations of heat flux.

5. Effect of the confinement

The structure of convection, the scalings for the heat flux and the thermal wind present a picture of convection within the TC at low Ekman numbers that resembles convection in a cylinder confined by solid walls: the steep rise heat transfer at low criticality, where the flow is dominated by rotation is followed by a rotation-independent regime at higher criticality. The similarity with rotating convection in a solid cylinder reflects a prominent role of background rotation through the TP constraint it imposes on the flow. Based on these observations, we shall conclude this study by measuring the degree of confinement within the TC. Figures 12a and 12c respectively show the z -rms profiles of vertical and

radial velocities for $E = 1.15 \times 10^{-5}$. The radial velocity right across the side boundary of the TC never exceeds a few percent of its typical value within the TC so the TP constraint can be seen as enforcing a near-impermeable condition there. For low criticality, vertical fluid motion is entirely contained within the radius of the TC. However, for $R_c \geq 6.41$, a slight motion appears outside the TC, that remains at approximately the same level as R_c is increased. The same is true for the azimuthal wind, which extends slightly beyond the TC at moderate to high levels of criticality (figures 10a and 10c). Vertical and azimuthal motions do not, however, directly break the TP constraint. Momentum inside the TC is indeed transported across the TC boundary by viscous friction. Friction is all the more effective there as a free Stewartson layer is expected to develop there, with an inner thickness scaling as $E^{-1/3}$ and an outer thickness scaling as $E^{-1/4}$ (Stewartson 1957). Such layers are too thin to be reasonably detected in our measurements. Nevertheless, the radial profiles of azimuthal velocities in figure 10a- 10d show hint of a variation in slope across the TC boundary. This local slope is also considerably steeper at $E = 6.36 \times 10^{-6}$ than $E = 1.15 \times 10^{-5}$. This suggests that the mechanical condition across the TC boundary is probably closer to one of imposed tangential stress of a value determined by the rotation, rather than the no-slip condition of a rigid boundary.

This tendency is confirmed by the z -rms profiles of vertical and radial velocities for $E = 6.36 \times 10^{-6}$. For this faster rotation, the radial velocity is found to be exactly zero at the boundary of the TC at all levels of criticality, within the precision of our measurements. Vertical velocity is also practically zero. The convection even seems extinct on the last 20% of the TC's outer region. Clearly, this behaviour is promoted by the fact that the hot boundary of the domain is itself confined within the domain. However, the intense convection that ensues would not remain confined within the higher latitudes of the TC without a strong influence of the TP constraint. Overall, the behaviour is similar to that found at $E = 1.15 \times 10^{-5}$, except that the TP constraint is more strictly enforced at equivalent levels of criticality.

Note that the curvature of the dome may have an influence on the confinement effect. The height under the dome varies by 7.8% between the centre and the edge of the TC. This geometry has two consequences: first, the onset of convection is determined by $RaE^{4/3}$, which depends on the local height under the dome h as $h^{1/3}$. This, however, implies that $RaE^{4/3}$ only varies by 2.5% across the heater which is unlikely to decide where the first convective cells appear. The limited influence of the curvature is further confirmed by the fact that the critical Rayleigh number follows practically the same scaling as for plane convection.

Second, the associated TP constraint opposes radial motion which slightly reinforces confinement. In planetary cores, the spherical shape of the solid core reverses the radial variation of height compared to our experiment but it similarly opposes radial motion, thus reinforcing confinement. In both cases, however, the confinement associated to the curvature of the dome is expected to be of significantly less influence than the TP constraint incurred at the outside edge of the TC.

6. Conclusions and discussion

The experimental study we conducted was focused on convection in a Tangent Cylinder for Ekman numbers in the range 3.36×10^{-6} to 4.51×10^{-5} , and brought several answers to the four questions set in the introduction:

First, the critical scalings for the onset of convection in a TC are similar to those

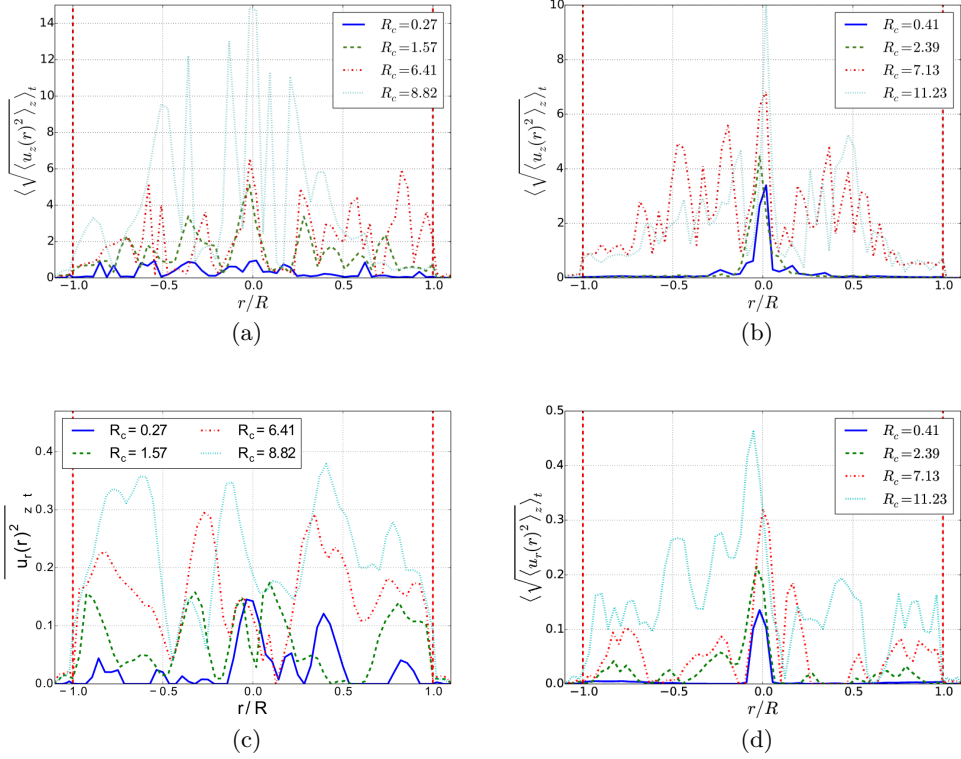


Figure 12: (a) and (c) $\langle \sqrt{u_z(r)^2} \rangle_t$ (top) and $\langle \sqrt{u_r(r)^2} \rangle_t$ (bottom) for several increments of supercriticality and $E = 1.15 \times 10^{-5}$. (b) and (d) Similarly, $\langle \sqrt{u_z(r)^2} \rangle_t$ and $\langle \sqrt{u_r(r)^2} \rangle_t$ for several increments of supercriticality and $E = 6.36 \times 10^{-6}$.

known for plane convection, albeit with different constants: the critical Rayleigh number $Ra_c = (26 \pm 4) \simeq E^{-4/3 \pm 0.1}$ is marginally higher, but the critical wavenumber $a_c = (0.5 \pm 0.07) \times E^{-1/3 \pm 0.05}$ is significantly higher than for plane convection. Second, this discrepancy has origins in the structure of the critical convective plumes, which resemble those found in rotating cylinders, rather than the periodic cell pattern of plane rotating convection. As in the former, the critical mode is either one of the slow or one of the fast modes identified by Goldstein *et al.* (1993), depending on the Ekman number and the aspect ratio of the cylinder. In a solid cylinder, the onset of wall modes takes place at a significantly lower critical Rayleigh number than the unstable modes of convection in a plane layer. In a Tangent Cylinder, however the Taylor-Proudman constraint does not exist in the still base flow, so the confinement that is responsible for the onset of wall modes is absent at such low values of Ra . Without it, the base flow retains a configuration that is stable up to values of Ra for which plane modes are unstable. When these are ignited, however, the TP constraint becomes active and favours eigenmodes of the cylindrical geometry. Nevertheless, the influence of the virtual TC boundary being weaker than that of a solid wall, the plumes at onset retain some of the features of that of a plane configuration, with in particular, a size that is intermediate between those of plane and cylindrical geometries.

Third, the loss of translational symmetry in the horizontal plane excludes an onset of

convection through steady modes, even at the moderate values of Pr for which one would expect steady rather than oscillatory onset in a plane configuration. However, the time-dependence takes the form of a very slow retrograde precession, instead of waves expected at oscillatory onsets.

Fourth, the supercritical regimes exhibit a complex sequence of convective patterns, leading to a single, large vortex centred on the cylinder axis when the Rayleigh number exceeds approximately 10 times the critical value. At relatively high latitude, this vortex translates into a coherent, retrograde thermal wind. For the Ekman numbers we considered, the intensity of the thermal wind, measured in terms of the Rossby number obeys a scaling of $Ro = (5.33 \pm 0.3) \times (Ra_q^*)^{0.51 \pm 0.04}$ identical to the scaling found in previous studies (Aurnou *et al.* 2003) for thermal wind resulting from the interplay between Coriolis, buoyancy force.

This phenomenology is supported by the scaling for the heat flux $Nu^* = (0.0046 \pm 0.0005) \times (Ra_q^*)^{0.26 \pm 0.04}$, which is also found in this regime.

Fifth, the geometry of the flow in the vicinity of the TC lateral boundary confirms that the Taylor-Proudman constraint is practically not broken there. On the other hand, for intense enough convection, the likely Stewartson layers that develop along this boundary diffuse a small part of the momentum generated inside the TC towards regions outside it.

Since our work has been largely motivated by the study of planetary cores, it is tempting to try and gain insight into their dynamics from these conclusions. This endeavour, however, meets several important obstacles. The first is the difference in the properties of the working fluids. Recently revised estimates of outer core thermal conductivity (de Koker *et al.* 2012; Pozzo *et al.* 2012) suggest that the Prandtl number could have a low value $Pr \sim 10^{-2}$. The onset of rotating convection at such Pr would be oscillatory even in the plane configuration. Nevertheless, it is still reasonable to expect that confinement within the TC induced by the virtual boundaries raised by the TP constraint reshapes convection in a similar way as it does at the moderate Prandtl numbers considered in this paper. This view is supported by the linear stability analysis of convection in a rotating cylinder at low Prandtl numbers by Goldstein *et al.* (1994), which shows that the critical modes are subject to both precession and oscillations. Furthermore, convection in the Earth core is also compositional in nature, rather than only thermal. The corresponding Schmidt numbers are in a range comparable to the thermal Prandtl numbers of water and acid, for which the onset of convection would not be oscillatory. Still, a deeper understanding of convection in a TC at low Prandtl number would require liquid metal experiments in a configuration similar to the present paper. This would clarify the question of whether the oscillatory modes at the onset of low- Pr convection are as robust to a change of boundary condition at the lateral boundary of the cylinder as we found the modes of moderate- Pr convection to be.

Secondly, several examples of nonmagnetic spherical shell convection driven by moderately supercritical convection of thermal (Sreenivasan & Jones 2006) or double-diffusive (e.g. (Trümper *et al.* 2012)) origin within the TC show an ensemble of thin viscously controlled plumes with no indication of any coherent z -vorticity. By contrast, coherent anticyclonic vorticity within the TC has been noted in rotating dynamo simulations (Glatzmaiers & Roberts 1995; Sreenivasan & Jones 2006; Schaeffer *et al.* 2017), which points to a possible role of the TC magnetic field in generating strong polar vortices in moderately supercritical convection. That said, the central retrograde vortex noted in our experimental TC occurs at $R_c \sim 10$, which might correspond to values of $R_c \sim 50$ –100 outside the spherical shell TC, given that convection inside the TC sets in at a Rayleigh

number much higher than outside it (Jones 2007). This strongly driven regime has not been adequately explored, at least in the computationally demanding regime of low E . Since the criticality of convection within Earth's TC is not well constrained, a comparison of the convection pattern at $R_c \sim 10$ within our experimental TC with that in the spherical shell TC would help ascertain whether the two patterns bear any resemblance to each other.

Finally, understanding the role of the Lorentz forces within the Earth's TC is not as straightforward as one might imagine. With a uniform magnetic field, it is known that the Lorentz forces favour large-scale magnetically controlled plumes in a plane layer (e.g. Aujogue *et al.* 2015). However, geodynamo simulations suggest that the mean field within the TC has severe lateral and axial inhomogeneities, whose effect on the width of convection plumes needs to be understood. Further, the nature of the free shear layers at the cylinder boundary may change as a consequence of the constraints that the Lorentz force has to satisfy to ensure sufficient smoothness of the velocity field (Livermore & Hollerbach 2012; Hollerbach 1994). It is hoped that magnetoconvection experiments in a TC configuration at moderate Prandtl number (which our apparatus is designed to perform) could provide us with a somewhat more realistic picture of convection within the TC than rotating convection alone.

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